Replication Training Plan

The following is an overview of what the heuristic method of teaching, as related to mathematics is, along with the four steps mentioned in the following paragraph. The goal of this training plan is to help you become familiar with what the heuristic teaching method in mathematics is, how to implement it in your classroom, and to have your students be involved in the mathematical thinking process.

Overview:

The **Heuristic Method** of teaching, in general, models the scientific method, and is broken into 4 main steps This method of teaching can be applied to just about any subject matter and is designed to engage the students in learning. The role of the teacher is to step back from their traditional role of direct instruction, and become a facilitator to clarify, explain, pose questions, suggest organizational and problem solving strategies, and mediate a class discussion, all the while resisting telling the students how to do something or just providing the answer. The teacher is there to guide them in the right direction so that they solve the problem, and will let them think through and figure the problem out on their own whenever possible. Thus, the idea is to ask questions to make sure they know what is going on, and become active in their learning. Asking the appropriate questions is key to this method. These questions can be specific questions to keep them thinking on the correct path, or less specific questions to generate discussion among the whole class. If the students start to stray from the correct method or reasoning, the teacher lets them. It is part of the exploring, but the teacher brings them back with questions for them to think about. After getting the correct answer, the teacher does something more than just telling the students that they are correct, the teacher poses follow up questions to test and challenge their understanding of the problem itself. All of these are in the 4 main steps that were mentioned earlier, and are listed below.

In our particular case, the heuristic method is applied to mathematics. As you and your students progress through the steps, you will ask the students questions, have them organize their thoughts, and think critically and logically to get to an answer. As many students tend to think that math has only one correct answer, it is important to emphasize that what is more important than the correct answer is the method and understanding of the problem. Such deeper understanding of the problem and the thought process is the core of the heuristic method.

**Step I: Wondering/ Read the Problem**

In this step, you ask the students, what do you want to discover? And what do you know? You ask them to make observations in mathematics and make connections with previous lessons and other areas of study as well. This is the stage in which they gather the given information, organize it, and make sure that they know exactly what the problem is asking for. Some of the things that you may ask and suggest them to do are as follows:
• What is known? Unknown?
• Have them note/highlight/circle important words.
• What do you want out of doing this problem?
• Restate the problem.
• Define any new vocabulary that they are uncertain about.

Here is a sample dialogue of Step 1 between a teacher and his/her class using the heuristic method.

Scenario: Today is the first introduction to inequalities. To illustrate an example, the instructor decides to let the students work through Example 2 on page 159 of their textbook using the Addition Property of Inequality. T stands for teacher and S stands for student, also (italics) are actions done by instructor. The following should be clearly written on the board so that everyone can read it.

Example 2 page 159. Using the Addition Property of Inequality.

Solve $7 + 3k > 2k - 5$.

T: “Okay class there is a problem for us to solve up on the board. We are going to tackle this problem just like we have the past several problems. Remember, we want to make this process a habit, and you should be doing all of this on your homework and practice problems. Write out each part of the steps. Let’s do this together to make sure everyone is following. What are we solving for in this problem? What’s the unknown?”

S: “We are solving for $k$, and that is our unknown in this problem.”

T: “Have we seen problems that look like this?” (While underlining the $k$ in the problem.) “Can we solve this problem? Is it fair for me to ask this problem on an exam?”

S: “No we haven’t, so no we can’t, and of course it isn’t fair if we haven’t seen it yet.”

T: “Let’s try and do what we have been methodically doing for all the past problems, and let’s try and apply the heuristic method to this problem. Is that reasonable?”

S: “Ok.”

T: “What does the symbol “>” mean? Does anyone know? How about you, Joe?”

S: “It means greater than, its on page 157 which we just went over.”

T: “So what is it exactly that we are looking for? If it were an equation with an “=” sign, we would know that we are looking for values of $k$ where our equation is satisfied. So what are we looking for in this example?”

S: “Values of $k$ where “>” is true.”
Step II: Designing/ Explore and Select a Strategy:
In this step you ask the students to gather the necessary information and make observations about the problem. You ask them how can you find out what we want? You engage them and ask them for problem solving strategies, and have them design strategies and discuss it among themselves. Some things that you may ask them to do or suggest are as follows:

• Picture the problem
• Organize the information
• Draw diagram/table and label as much as you can with colored markers if possible or applicable
• Think of a related problem that helps
• Look for patterns
• Work backwards
• Guess and check
• Reduce or simplify the problem first
• Find easier problems that are similar

At this point in the process, you want to make sure of the following:
• The students should have a clear understanding of the problem, the proposed process and the reasoning behind the process.
• You should take your time when explaining, and explain everything, and you should not assume that they know something which you may think is obvious.
• You should encourage the students to be engaged in the process, and try to have every student in your class involved.
• You should not pick on any of particular student, and if students make suggestions or say something that is incorrect, you should not shut them down. You want to let them try it out and see for themselves why what or how they answered is wrong/incorrect.
• Remind the students of this famous quote if they suggest something wrong: Edison once said, “If I find 10,000 ways that something won’t work, I haven’t failed. I am not discouraged, because every wrong attempt discarded is another step forward.”

Here is a sample dialogue of Step II

T: “Great job. Anyone have any ideas on how we should try and solve this problem? How does the “>” symbol affect the inequality?”

S: “I’m not sure, we haven’t seen this before. But if we compare it to a normal equation with an “=” sign, then we would move the 2k to the left, and the 7 to the right.”

T: “Can we do it with our current problem? What do you guys think? Any other suggestions?”
Pathways to STEM

S: “I’m not sure, we could guess and check…”

T: “We could do that, but it may take a while, so let’s come back to that suggestions in a little bit, but I’ll write it on the board for now. Make sure I come back to that process class. If there are no other suggestions, let’s try and use the first suggestion. So tell me what to write clearly and justify each step for me.”

**Step III: Investigating/ Solving**

In this step you ask the students to solve their problem with the information and strategies from Step II. Some of the things that you may suggest or ask them to do are:

- Implement your strategies from Step II, using your information from Step II.
- Think of big steps then do little steps.
- Use another strategy if the 1st one doesn’t work.
- Look back and solve a similar or easier problem.
- Have your students write everything explicitly on their paper. This way they can refer to it later if they need to.
- Tell your students to have someone check their work (if it’s not an exam).
- Tell your students to be critical and constructive when checking each other’s work.
- Remind the students that math is creative, never give up, and there are often more ways than one to solve the same problem.

**Here is a sample dialogue of Step III**

S: “Considering that we are treating these as a normal equation with an “=” sign, the we can move the $2k$ to the left by subtracting $2k$ from both sides, and same thing for the 7, subtract 7 from both sides.”

T: “Ok class, great, so what do we have left?”

S: “We end up with $k > -12$.”

**Step IV: Discovery/ Looking Back**

In this step of the heuristic method, you ask the students what they discovered and what did they learn. You want them to be able to state their answer, and determine if the answer that they obtained is correct or not, but more than that, you want them to demonstrate to you that they have learned. This can be done by not only checking their steps but why they took such steps. Some things that you may ask them to do, think about, and show are the following:

- Check answer(s).
- Does it make sense?
- Reflect on the solution, and does it answer the original question.
- The process used is key, not the answer itself.
- Watch for careless errors.
- Have them ready to justify everything they did and be able to explain it to someone else to test their understanding.
• Suggest they share their process and solutions with others, and find alternate processes. Remind that many times in math, there is more than one way to do a problem.
• Ask them questions to make sure they understand the problem. i.e. what would happen if the problem had a slight variation, would the same process work? If yes, try it out and do more exploring of the problem.

Here is a sample dialogue of Step IV

T: “Does it make sense? Is there a way we could check? How can we be sure that we didn’t make any careless errors?”

S: “I’m not sure how we could check or see if we have careless errors.”

T: “How did we check if our solutions to equations with ‘=’ were correct?”

S: “We put in the solution and see if both sides were ‘=’.”

T: “Ok….. can we try that strategy here?”

S: “Yes! We can just pick numbers less that -12 to make sure that our statement is correct.”

T: “Good job, so lets all be active learners and each choose a number less that -12 and share our findings. I’ll give you guys a minute to do this.” (After a minute.) “So what do we have? Does our statement $k > -12$ hold true?”

S: “Yes it does.”

T: “Great good job guys. So what happens at 0? At 5? At -12? Is the statement true?”

S: “At 0 and 5 it’s true, but at -12 we have equality.”

T: “So in our interval notation, should there be a bracket or parenthesis, should our graph have an open or filled in circle?”

S: “Open and open!”

T: “What about the guess and check method? Would it work here? How could we have used it if possible?”

S: “It would have been longer if we didn’t know where to start since there are a lot of numbers to start with, but maybe we could have started by finding where the inequality is equal first then check from there. So it is a valid way to attempt the problem.”

T: “What if I flip the sign, i.e. turn ‘>’ into ‘<’?”
S: “Same method works, just the sign in the final solution will be flipped.”

T: “Good job class, lets do several more for more practice, and keep in mind the heuristic process. And this is now a fair question for the exam, it always was, you all just solved the whole thing, I didn’t do anything but write your ideas on the board.”

Now you are going to look at teaching using this method in a classroom. But first let’s review some of the benefits and challenges you may encounter using this method.

Benefits of the Heuristic Teaching Method in Mathematics:

- Students become more familiar with each other and feel more comfortable asking questions.
- Sometimes students feel more comfortable talking to each other than the Professor or Lecturer.
- Overall better learning environment throughout the course.
- Learning of effective problem solving methods, which are applicable to all technical and scientific fields.

Challenges in Using the Heuristic Method

- Lack of student initiatives. Some students feel it is much more comfortable and easy to just listen to a lecture and take notes rather than be questioned, “What do you want to learn about this subject, and how do you propose to learn it?”
- If the method is not taken seriously by the students, and they are not involved, the whole method falls apart. The method relies on the active participation of the students.
- It is also a time-consuming method than the direct teaching until students become more familiar with the method.
- A few students may dominate the question answering if they know the material. This will not help other students as much.

Instituting the Heuristic method in your classroom

The idea is to periodically use this method of teaching along side your normal classroom lectures or whichever method of teaching you do. This method could be used to obtain feedback of previous lessons that your students have already seen, reinforcing old material, or even possibly introducing new material that builds upon previous lessons. As indicated above, the role of the lecturer should be as minimal as possible.

Steps:
1. Consider the size of your classes. If your class size is less than 10, the entire class may participate as one group, or break them into smaller groups. The reason for
the groups is so that you can give them some time to work on it in their groups, and then make them present to the class. Sometimes it may be easier to handle the smaller groups, and it may help the students become more comfortable in sharing. Another thing you can do is give each group a different problem and have them follow the steps and present each step to the class. Remember, the important thing here is the process, not the necessarily the correct answer. Your goal with the heuristic method is to train students to think, so that they can adapt and follow the guidelines to solve other problems on their homework or exams.

a. Give a problem to the class, write it out clearly on the board or clearly reference it somewhere readily available for them to look at such as a problem from a book or handout.

b. For the Wondering Step, ask them the questions and have them follow the bullets in this step. They should, at the least, have an idea of what they are solving for. If it’s a word problem, try and relate it to something they can visualize readily so that they have a better understanding, and see more motivation or purpose for learning the material.

c. Ask the students questions to make sure they know what they are solving for. The reason for asking questions serves several purposes. The first is that you get feedback to see if they understand the problem, and the second is so that you can stimulate and guide their thinking without telling them how to do the problem. You may call on the students individually, or ask for volunteers to relate the problem to something concrete or something that they have already seen. Asking them to explain it to the class is another good thing for students to do as it supports their learning from each other, and test their individual understanding as well. Remember to try not to call on the same students, and try to encourage them all to share as much as possible. You are trying to create a learning atmosphere where they feel comfortable and can learn not just the math, but also the process. (Remember you are just there to ask and guide them through the process. You are not there to give them answers or walk them through the math. You are there to walk them through the process of thinking, and they should tell you the math.)

d. For the Designing Step, follow the suggestions for the bullets. Ask them to help you draw a picture, or describe something that they can relate it to. Whatever connection they can make to help them better understand the problem the better off they are. Have them tell you what to label, and how to set the problem up. Do exactly as they say, not as what you think they mean, otherwise they may be getting too much help. Let them introduce the variables and other steps that need to be used. Ask them if they notice anything about the problem, if it’s similar to a previous problem, if they see patterns, etc. Also, what strategies they think would work. List these on the board to come back to them later.
e. The Investigating Step is pretty straightforward, just remember to let the students tell you what to do, and do exactly what they tell you to do. It will also help them if they use the math terms – e.g. state the numerator and denominator vs. the number onto and the number on the bottom. Go back to their list on the board and go over a couple of their suggestions. This will show them that often times there is more than one way to solve something, and will give them more insight and creativity in their own work. Never just dismiss a suggestion, try to explain why it won’t work, or work through it and let them see why it won’t work. If none of their suggestions work, then ask them questions to steer them into the correct train of thought, and as much as possible, provide the bare minimum of assistance. As a last resort, you can explain to them how to solve the problem, but ask them to explain it back to you and justify their steps and thought process.

f. The Discovery Step is straightforward. Have them reflect on the first 3 steps and let them think about their answer. Make sure it makes sense to them, and their answer seems reasonable. Now you can really test to see if they understand the problem. You may ask questions that alter the problem slightly and ask them how it would affect the solution. Or maybe ask them to find a general formula or something with letters instead of numbers.

Some reminders:
To obtain the best results using the heuristic method, you should:

- Encourage the questioning mind, and equip students with skills for finding the answers.
- Train the students to think and follow the process.
- Insist that the students provide evidence and make a convincing case for what they think and say.
- Tell the students to be methodical and lead by example when completing example problems.
- Explain every step of the process and make sure that they are following along.
- Engage students in the process and ask them questions to make sure they know what exactly is going on.